647510 **BSCmaMN353B-6C** **Seat No :________**

B.Sc. Semester - 6 (NEP-2020) Examination

March/April-2026

MATH: Vector Calculus and Multiple Integrals

(For students having Major subject other than Mathematics) (Minor-6)

Time: 2:00 Hours **Marks: 50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any two out of the following. (10)

1. Prove: $\hat{\imath} \times (\vec{a} \times \hat{\imath}) + \hat{\jmath} \times (\vec{a} \times \hat{\jmath}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\,\vec{a}$.
2. Prove the identity: $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$
3. Explain the geometrical interpretation of scalar triple product. If $\vec{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}, \vec{b} = -3\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$ and $\vec{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$ then find its scalar triple product.

Que.2 Answer any two out of the following. (10)

1. Define Scalar field and vector field and give one example of each. Also define ∇ operator.
2. Define curl of $\bar{f}$. Find $\nabla \times \bar{f}$ where $\bar{f} = x^3y^2z\hat{\imath} + xy^3z^2\hat{\jmath} + x^2yz^3\hat{k}$ at (1,2,3).
3. If $r = |\bar{r}|$ where $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then prove the following.

   (i) $\nabla f(r) = f'(r)\frac{\bar{r}}{r}$ (ii) $\nabla f(r) \times \bar{r} = 0$

Que.3 Answer any two out of the following. (10)

1. Evaluate $\iint \sqrt{xy(1-x-y)}\ dx\,dy$ over the area of triangle bounded by lines x=0,y=0, x+y=1.
2. Find the volume of a sphere using multiple integral.
3. Prove: $\iint_D \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\ dx\,dy = \frac{\pi}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right)$, where D is the third quadrant of the unit circle.

Que.4 Answer any two out of the following. (10)

1. Evaluate: $\oint_c \bar{F} \cdot d\bar{r}$, where $\bar{F} = (x - 3y, 2x - y)$ and C is an ellipse $4x^2 + 9y^2 = 36$.
2. Verify Green's theorem for the line integral $\oint_C (x-y)^3dx + (x-y)^3dy$, where C is a circle $x^2 + y^2 = a^2$.
3. Show that the vector field $\vec{F} = 2x(y^2 + z^3)\hat{\imath} + 2x^2y\hat{\jmath} + 3x^2z^2\hat{k}$ is conservative. Find its scalar potential and work done in moving a particle from $(-1,2,1)$ to $(2,3,4)$.

Que.5 Answer any two out of the following. (10)

1. If $\bar{V} = y\hat{\imath} + z\hat{\jmath}$ and S is a part of $2x + 2y + z = 2$ then find $\iint_s V_n\, d\sigma$.
2. Evaluate: $\iiint_R x^2\, dx\, dy\, dz$, where R is a closed surface of cylinder bounded by z=0, z=2 and $x^2 + y^2 = 9$.
3. Prove that $div(r^n\bar{r}) = (n+3)r^n$.

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